Test 3 Numerical Mathematics 2
November, 2021

Duration: 6 quarters of an hour.

In front of the questions one finds the points. The sum of the points plus 1 gives the end mark of this test. Use of a calculator is allowed.

1. [1] Show that given an innerproduct (f, g) one can generate orthogonal polynomials $\phi_n(x)$ of degree n by a triple-recursion relation, when the two polynomials $\phi_0(x)$ and $\phi_1(x)$ are given.

2. Consider the function $f(x)$ on [-1,1] given by

$$f(x) = \begin{cases} 1 + x & \text{for } x \in [-1, 0], \\ 1 - x & \text{for } x \in [0, 1]. \end{cases}$$

   (a) [1.5] Let $C_n(x) = \sum_{k=0}^{n} a_k T_k(x)$ be the Chebyshev expansion of $f(x)$. Give the formula for a_k and use this to show $a_k = 0$ for k odd.

   (b) [2.5] Compute a_0 and a_2. For the latter you might need one of $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$ or $\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$.

3. Consider for arbitrary $f(x)$ the integral $\int_0^\infty w(x)f(x)dx$ with $w(x) = \exp(-2x)$.

   (a) [3] Derive that for this integral the Gauss rule using one interpolation point is simply $f(1/2)/2$.

   (b) [1] Determine the degree of exactness of the rule in the previous part.